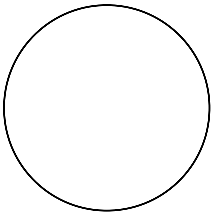
Too many variants in equilibrium?

A model without location choice.

Focus on firms' entry into the market.

The circular city



Circumference: 1
Consumers uniformly distributed around the circle.
Number of consumers: 1
Linear transportation costs: $t(d) = td$
Unit demand, gross utility = s

Entry cost: f

Unit cost of production: c

Profit of firm i: $\pi_i = \begin{cases} (p_i - c)D_i - f, & \text{if it enters,} \\ 0, & \text{otherwise} \end{cases}$

Two-stage game:

Stage 1: Firms decide whether or not to enter. Assume entering firms spread evenly around the circle.

Stage 2: Firms set prices.

If n firms enter at stage 1, then they locate a distance $1/n$ apart.

Stage 2: Focus on symmetric equilibrium.

If all other firms set price p, what then should firm i do?

Each firm competes directly only with two other firms: its neighbours on the circle.

At a distance $\tilde{x}$ in each direction is an indifferent consumer:

$$p_i + t\tilde{x} = p + t\left(\frac{1}{n} - \tilde{x}\right)$$

$$\tilde{x} = \frac{1}{2t}\left(p + \frac{t}{n} - p_i\right)$$

Demand facing firm i:

$$D_i(p_i, p) = 2\tilde{x} = \frac{1}{n} + \frac{p - p_i}{t}$$

Firm *i*’s problem:

$$\max_{p_i} \pi_i = (p_i - c)\left(\frac{1}{n} + \frac{p - p_i}{t}\right) - f$$

$$\frac{\partial \pi_i}{\partial p_i} = \left(\frac{1}{n} + \frac{p - p_i}{t}\right) - (p_i - c)\frac{1}{t} = 0$$

$$2p_i - p = c + \frac{t}{n}$$

In a symmetric equilibrium, all prices are equal. $\Rightarrow p_i = p$.

$$p = c + \frac{t}{n}$$

Stage 1:
How many firms will enter?

$$D_i = \frac{1}{n}$$

$$\pi_i = (p - c)\frac{1}{n} - f = \frac{t}{n^2} - f$$

$$\pi = 0 \Rightarrow n = \sqrt{\frac{t}{f}}$$

$$\Rightarrow p = c + \frac{t}{\sqrt{t/f}} = c + \sqrt{tf}$$

Condition: Indifferent consumer wants to buy:

$$s \geq p + \frac{t}{2n} = c + \frac{3}{2}\sqrt{tf} \iff f \leq \frac{4}{9t}(s-c)^2$$

Exercise 7.3: What if transportation costs are quadratic?

[Exercise 7.4: What if fixed costs are large?]

<u>Social optimum</u>: Balancing transportation and entry costs.

Average transportation cost: $t\,\frac{1}{2}\tilde{x} = \frac{t}{2}\frac{1}{2n} = \frac{t}{4n}$

The social planner's problem:

$$\min_n \left(nf + \frac{t}{4n} \right)$$

FOC: $f - \frac{t}{4n^2} = 0 \quad \Rightarrow n^* = \frac{1}{2}\sqrt{\frac{t}{f}} < n^e$

Too many firms in equilibrium.

Private motivation for entry: business stealing
Social motivation for entry: saving transportation costs

[Exercise: What happens with n^e/n^* as N (number of consumers) grows?]

Advertising

- informative
- persuasive

Persuasive: shifting consumers' perferences?

Focus on informative advertising.

Hotelling model, two firms fixed at 0 and 1, consumers uniformly distributed across [0,1], linear transportation costs td, gross utility s.

A consumer is able to buy from a firm if and only if he has received advertising from it.

φ_i – fraction of consumers receiving advertising from firm i

Advertising costs: $A_i = A_i(\varphi_i) = \frac{a}{2}\varphi_i^2$

Potential market for firm 1: φ_1.
Out of these consumers, a fraction $(1 - \varphi_2)$ have not received any advertising from firm 2.
The rest, a fraction φ_2 out of φ_1, know about both firms.

Firm 1's demand:

$$D_1 = \varphi_1[(1 - \varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)]$$

A simultaneous-move game.

Each firm chooses advertising and price.

Firm 1's problem:

$$\max_{p_1,\varphi_1} \pi_1 = (p_1 - c)\varphi_1\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - \frac{a}{2}\varphi_1^2$$

Two FOCs for each firm.

$$\text{FOC}[p_1]:\ \varphi_1\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - (p_1 - c)\frac{\varphi_1\varphi_2}{2t} = 0$$

$$\text{FOC}[\varphi_1]:\ (p_1 - c)\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - a\varphi_1 = 0$$

$$\Rightarrow \quad p_1 = \frac{1}{2}(p_2 + c - t) + \frac{t}{\varphi_2}$$

$$\varphi_1 = \frac{1}{a}(p_1 - c)\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right]$$

Firms are identical $\Rightarrow$ Symmetric equilibrium

$$p = \frac{1}{2}(p + c - t) + \frac{t}{\varphi}$$

$$\Rightarrow \; p = c + t\left(\frac{2}{\varphi} - 1\right)$$

$$\varphi = \frac{1}{a}(p - c)\left[(1 - \varphi) + \varphi\frac{1}{2}\right]$$

$$\varphi = \frac{1}{a}t\left(\frac{2}{\varphi} - 1\right)\left(1 - \frac{\varphi}{2}\right)$$

$$\Rightarrow \; \varphi = \frac{2}{1 + \sqrt{\frac{2a}{t}}}$$

Condition: $\frac{a}{t} \geq \frac{1}{2}$

$\Rightarrow p = c + \sqrt{2at}$

Condition: $s \geq c + t + \sqrt{2at}$ $(\geq c + 2t)$

- $\frac{\partial \varphi}{\partial a} < 0, \quad \frac{\partial p}{\partial a} > 0$

Firms' profit:

$$\pi = \frac{2a}{\left(1+\sqrt{\frac{2a}{t}}\right)^2}$$

- $\frac{\partial \pi}{\partial t} > 0; \quad \frac{\partial \pi}{\partial a} > 0!$

An increase in advertising costs increases firms' profits.

Two effects of an increase in *a* on profits:

A direct, negative effect.
An indirect, positive effect: $a\uparrow \;\rightarrow\; \varphi\downarrow \;\rightarrow\; p\uparrow$

Firms profit collectively from more expensive advertising.

Crucial assumption: convex advertising costs.

What about the market for advertising?
[Kind, Nilssen & Sørgard, 2007, 2009]

Social optimum

Average transportation costs

among fully informed consumers: $t/4$.

among partially informed consumers: $t/2$.

The social planner's problem:

$$\max_{\varphi} \varphi^2\left(s-c-\frac{t}{4}\right)+2\varphi(1-\varphi)\left(s-c-\frac{t}{2}\right)-2\frac{a}{2}\varphi^2$$

$$\varphi^* = \frac{2(s-c)-t}{2(s-c)+2a-\frac{3}{2}t}$$

[Condition: $t \leq 2(s-c)$]

Special cases:

(i) $\frac{a}{t} \to \frac{1}{2}$:

$$\varphi^e \to 1$$

$$\varphi^* \to 1-\frac{t}{4(s-c)-t} < 1$$

Too much advertising in equilibrium

(ii) $\frac{a}{t} \to \infty$:

$$\varphi^e \to 0$$

$$\varphi^* \to \frac{1}{1+\frac{a}{s-c}} > 0$$

Too little advertising in equilibrium

Vertical product differentiation

Quality competition

Consumers agree on what is the best product variant.
But they differ in their willingness to pay for quality.

s – quality
θ – measure of a consumer's taste for quality.

If a consumer of type θ buys a product of quality s at price p, her net utility is:

$$U = \theta s - p$$

$F(\theta)$ – cumulative distribution function of consumer type

$F(\theta')$ – fraction of consumers with type $\theta \leq \theta'$.

Unit demand: If $\theta s - p \geq 0$, then a consumer of type θ buys one unit of the good.

One firm:

At price p, its demand is $D(p) = 1 - F\left(\frac{p}{s}\right)$.

Two firms:

Suppose $s_1 < s_2$, $p_1 < p_2$. The indifferent consumer:

$$\tilde{\theta}\, s_1 - p_1 = \tilde{\theta}\, s_2 - p_2$$

$$\tilde{\theta} = \frac{p_2 - p_1}{s_2 - s_1}$$

Product 2 *quality dominates* product 1 if:

$$\tilde{\theta} < \frac{p_1}{s_1} \Leftrightarrow \frac{p_2}{s_2} < \frac{p_1}{s_1}$$

Otherwise $\left(\frac{p_2}{s_2} \geq \frac{p_1}{s_1}\right)$, demand is:

$$D_1(p_1, p_2) = F\left(\frac{p_2 - p_1}{s_2 - s_1}\right) - F\left(\frac{p_1}{s_1}\right)$$

$$D_2(p_1, p_2) = 1 - F\left(\frac{p_2 - p_1}{s_2 - s_1}\right)$$

Assume:

Consumers uniformly distributed across $[\underline{\theta}, \overline{\theta}]$

Consumers sufficiently different:

$$\overline{\theta} > 2\underline{\theta}$$

(avoiding quality dominance in equilibrium)

Firm 2 is the high-quality producer: $s_2 > s_1$.

Production costs independent of quality: c

Equilibrium in prices

$$\tilde{\theta} = \frac{p_2 - p_1}{s_2 - s_1}$$

Firm 1's profit: $\pi_1 = (p_1 - c)\left(\frac{p_2 - p_1}{s_2 - s_1} - \max\left[\underline{\theta}, \frac{p_1}{s_1}\right]\right)$

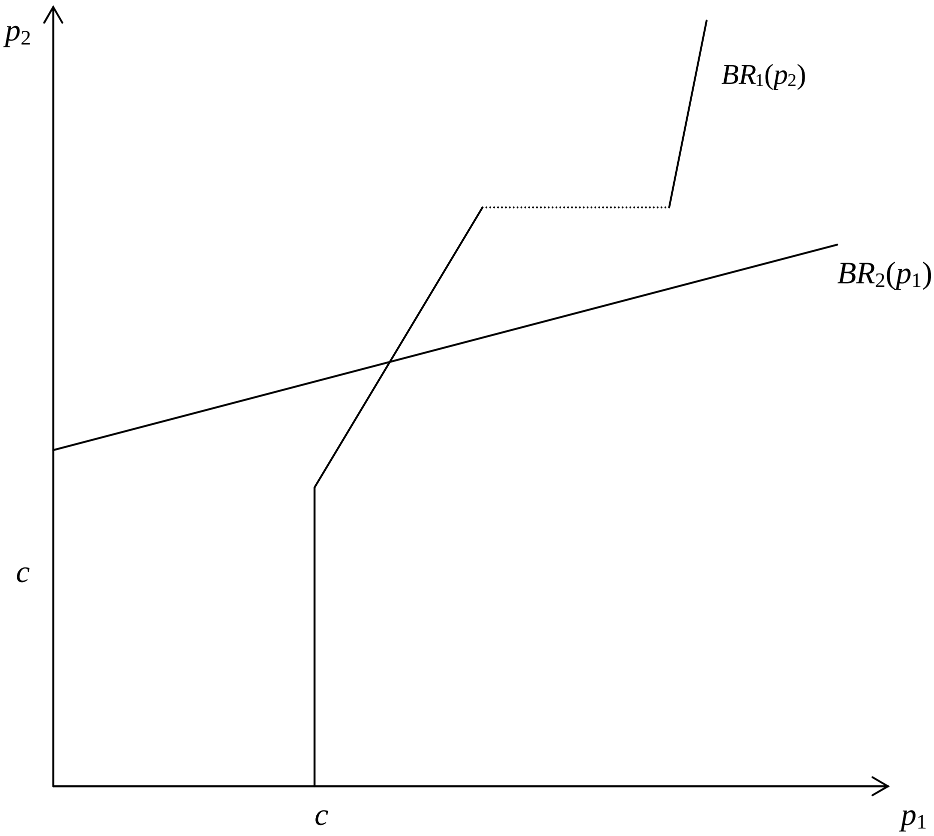
Best response of firm 1:

$$p_1 = \begin{cases} \frac{1}{2}\left[c + \frac{s_1}{s_2} p_2\right], \text{ if } p_2 > c + \underline{\theta}(s_1 + s_2) \\ \frac{1}{2}\left[c + p_2 - \underline{\theta}(s_2 - s_1)\right], \text{if } c + \underline{\theta}(s_1 + s_2) \geq p_2 \geq c + \underline{\theta}(s_2 - s_1) \\ c, \text{ if } p_2 < c + \underline{\theta}(s_2 - s_1) \end{cases}$$

Firm 2's profit: $\pi_2 = (p_2 - c)\left(\overline{\theta} - \frac{p_2 - p_1}{s_2 - s_1}\right)$

Best response of firm 2:

$$p_2 = \frac{1}{2}\left[c + p_1 + \overline{\theta}(s_2 - s_1)\right]$$



Equilibrium prices:

$$p_1 = c + \frac{1}{3}\left(\overline{\theta} - 2\underline{\theta}\right)\left(s_2 - s_1\right)$$
$$p_2 = c + \frac{1}{3}\left(2\overline{\theta} - \underline{\theta}\right)\left(s_2 - s_1\right)$$

Condition for the market being *covered*, $\underline{\theta} \geq \frac{p_1}{s_1}$:

$$c \leq \tfrac{1}{3}[\underline{\theta}(2s_1 + s_2) - (\overline{\theta} - \underline{\theta})(s_2 - s_1)]$$

- The high-quality firm sets the higher price:

$$p_2 - p_1 = \tfrac{1}{3}(\overline{\theta} + \underline{\theta})(s_2 - s_1) > 0$$

- The high-quality firm has the higher demand:

$$\tilde{\theta} = \frac{p_2 - p_1}{s_2 - s_1} = \tfrac{1}{3}(\overline{\theta} + \underline{\theta}) < \tfrac{1}{2}(\overline{\theta} + \underline{\theta})$$

$$D_1 = \tilde{\theta} - \underline{\theta} = \tfrac{1}{3}(\overline{\theta} - 2\underline{\theta})$$

$$D_2 = \overline{\theta} - \tilde{\theta} = \tfrac{1}{3}(2\overline{\theta} - \underline{\theta})$$

- The high-quality firm has the higher profit:

$$\pi_1(s_1, s_2) = (p_1 - c)D_1 = \tfrac{1}{9}(\overline{\theta} - 2\underline{\theta})^2(s_2 - s_1)$$

$$\pi_2(s_1, s_2) = (p_2 - c)D_2 = \tfrac{1}{9}(2\overline{\theta} - \underline{\theta})^2(s_2 - s_1)$$

- Firms' profits are increasing in the quality difference

<u>Two-stage game</u>

Stage 1: Firms choose qualities
Stage 2: Firms choose prices

Stage 1 – feasible quality range: $[\underline{s}, \overline{s}]$
Assume: $c \leq \tfrac{1}{3}[\underline{\theta}(2\underline{s} + \overline{s}) - (\overline{\theta} - \underline{\theta})(\overline{s} - \underline{s})]$

In equilibrium: $s_1 = \underline{s}$, $s_2 = \overline{s}$ (or the opposite).

- Asymmetric equilibrium
- Maximum differentiation

What if …

- $c > \frac{1}{3}[\underline{\theta}(2\underline{s} + \bar{s}) - (\bar{\theta} - \underline{\theta})(\bar{s} - \underline{s})]$
    - the low-quality firm will choose a quality above $\underline{s}$.

- $\bar{\theta} < 2\underline{\theta}$
    - only one firm active in the market:
      $p_1 = c$, $D_1 = 0$, $\pi_1 = 0$
      $p_2 = c + \frac{1}{2}\bar{\theta}(\bar{s} - \underline{s})$, $D_2 = 1$, $\pi_2 = \frac{1}{2}\bar{\theta}(\bar{s} - \underline{s})$
    - natural monopoly: low consumer heterogeneity makes price competition too intense for the low-quality firm

Natural duopoly for a range of consumer heterogeneity above $\bar{\theta} > 2\underline{\theta}$.

<u>Vertical differentiation:</u> the number of firms determined by *consumer heterogeneity*.

<u>Horizontal differentiation:</u> the number of firms determined by *market size*.